Washington Math Olympiad

Year 3

April 7, 2025

Welcome to the Washington Math Olympiad (WAMO), a contest dedicated to serving and strengthening the Washington math community.

Instructions:

- You will have **75 minutes** to solve **15 integer-answer problems**.
- There is **no penalty for guessing**.
- The only permitted aids are writing utensils, erasers, and blank scratch paper.
- Graph paper, calculators, and any other computing devices are **not** allowed.

1. Compute x given that

$$\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} = 93.$$

2. A certain school has 120 students. If $X\%$ of these students ride the bus to school, how many distinct integer values of X are possible?

3. Consider a circle centered at the origin. This circle is partitioned into four quadrants by the x and y axes. Within each quadrant, a circle is inscribed such that it is tangent to both the large circle and the corresponding axes. A fifth circle is externally tangent to all four of these inscribed quadrant-circles. Let the ratio of the area of this fifth circle to the area of the original large circle be R. If R can be expressed as $a - \sqrt{b}$, where a and b are positive integers, compute $a + b$.

4. Let $P_1P_2 \ldots P_{20}$ be a regular 20-gon. A subset of the vertices $P_1, P_2, \ldots, P_{20}$ is selected at random, with each non-empty subset of fewer than 20 vertices equally likely. The probability that every element within the chosen subset lies within a circle while all other vertices lie outside of the circle can be expressed as $\frac{m}{n}$, where m and n are positive integers with $\gcd(m, n) = 1$. Compute the sum of the positive factors of m.

5. A point M lies inside a circle. Two equilateral triangles $\triangle MPQ$ and $\triangle MRS$ are drawn such that points P, Q, R, S lie on the circle. If MPQ has side length 12, and MRS has side length 18, what is the square of the radius of the circle?

6. Let $a_n = 1^2 + 2^2 + 3^2 + \ldots + n^2$. Compute the number of positive factors of the sum $a_1 + a_2 + a_3 + \ldots + a_{100}$.

7. Compute the sum of all possible distinct values of the expression

$$(1 \pm 1)(1 \pm 1 \pm 1 \pm 1)(1 \pm 1 \pm 1 \pm 1 \pm 1),$$

where each $\pm$ independently represents a choice of $+$ or $-$.

8. The graph of the equation

$$x^2 + y^2 + 216 = 36(|x| - |y|)$$

consists of multiple curves. The total length of these curves can be expressed as $a\pi\sqrt{b}$, where a and b are positive integers and b is square-free. Find ab.

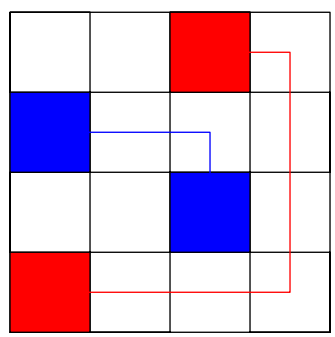
9. Let $N = (((1@4)@3)@4)$, where each @ can independently be addition, subtraction, multiplication, or division, each with equal probability. The expected value of N can be expressed as $\frac{m}{n}$, where m and n are positive integers and $\gcd(m, n) = 1$. Compute $m + n$.

10. A 11-digit arrangement of the set $\{0, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is called *amazing* if
    - 0, 1, 3, 4 appear in increasing order (not necessarily consecutively), and
    - 5, 6, 8, 9 appear in decreasing order (not necessarily consecutively).

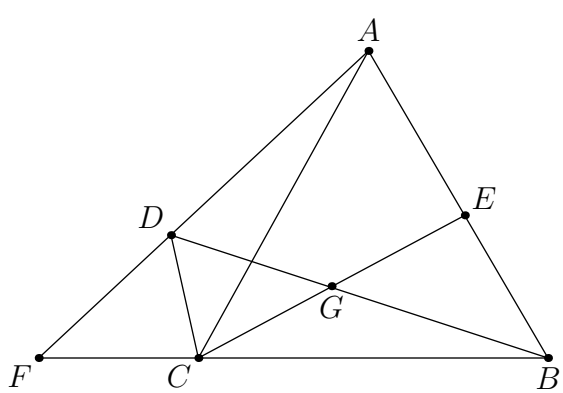
    For example, $9, 1, 8, 0, 1, 6, 7, 3, 2, 4, 5$ and $2, 9, 8, 0, 1, 6, 7, 3, 1, 4, 5$ are *amazing* arrangements. How many *amazing* arrangements are there?

11. Let a, b, c, be positive integers such that the expression $(a - \frac{1}{b})(b - \frac{1}{c})(c - \frac{1}{a})$ is also a positive integer. Compute the positive difference between the maximum and minimum possible values of the expression $7a + 8b + 9c$.

12. In a 4×4 grid of unit squares, two squares are colored red and two are colored blue. A configuration is called *connected* if both red squares are connected by a red path passing through the edges of the grid, and the blue squares are connected by a similar blue path. The red and blue paths, including the start and end squares, must not intersect. How many configurations are *not connected*?



13. As shown in the diagram below, D is a point outside equilateral triangle ABC with $\angle ADC = 120°$. Lines AD and BC are extended to meet at point F, and point E lies on segment AB such that $BE = CF$. Given that $DG = 3$ and $AF = 8$, compute the length of segment BD.



14. The value of

$$\sum_{k=2}^{9999} \frac{101 - \lceil \sqrt{k+2} \rceil}{k(k+1)}$$

can be expressed as $\frac{m}{n}$, where $\gcd(m, n) = 1$. Compute the remainder when $m+n$ is divided by 1000.

15. Let (x, y) be an ordered pair of real numbers satisfying the system of equations

$$\sqrt{x^2 + y^2 - 8x + 8y + 32} + \sqrt{x^2 + y^2 + 8x - 8y + 32} = 10\sqrt{2}$$
$$5(xy + 8)(x^2 - y^2) = 1224$$

Compute the sum of all distinct possible values of $x^2 + y^2$.